

Temperature in a glacier: Part I The Steady State

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Background

A glacier transports heat both through diffusion and also through its flow. That portion transported through flow is so substantial that no accurate analysis of internal temperature in a glacier is possible without considering it. A highly simplified model of flow, but one that captures the essence of heat transport nevertheless, is as follows.

Snow accumulates at the surface of the glacier. Its weight bears upon the snow below it, and causes a vertical compression or strain of the glacier. At the same time the glacier moves laterally. At the base of the glacier all vertical strain ceases and the motion is entirely lateral. A model that describes this motion is the complex velocity potential $f(w) = \frac{1}{2} \epsilon w^2$; where ϵ is the vertical strain rate of the glacier and w is the complex variable $x + jz$. At any location the vertical velocity of material is $-z$ where z is the height above the bedrock. Using this flow model, the 1-D equation for heat transfer is

$$1. \alpha \frac{\partial^2 T}{\partial z^2} + \epsilon z \frac{\partial T}{\partial z} = \frac{\partial T}{\partial t}$$

The first term describes conductive heat transfer, in which α is the thermal diffusivity of the ice. The second term describes convective heat transport. If strain rate, ϵ , is large enough, temperatures in the glacier will reach a steady state very quickly. In this case the differential equation governing temperature is found by setting the explicit time dependence equal to zero. Thus,

$$2. \alpha \frac{\partial^2 T}{\partial z^2} + \epsilon z \frac{\partial T}{\partial z} = 0$$

Solution of the problem

Let $P = \frac{\partial T}{\partial z}$, then the differential equation becomes of first order, $\alpha \frac{\partial P}{\partial z} + \epsilon z P = 0$. There are two distinct ways of looking at boundary conditions. In a truly steady state the surface temperature of the glacier is constant at a temperature of T_0 . At the base of the glacier one might specify a constant temperature, a constant gradient, or a mixture of the two.

In the case of a constant temperature at the base of the ice the solution is...

$$3. T(z) = (T_b - T_0) \left[1 - \frac{\text{Erf}(z\sqrt{\epsilon/\alpha})}{\text{Erf}(h\sqrt{\epsilon/\alpha})} \right] + T_0; \text{ where } T_b \text{ is the constant basal temperature.}$$

In the case of a constant gradient at the base of the glacier the solution is ...

4. $T(z) = T_o + G(\sqrt{\pi\alpha/4\epsilon})[\text{Erf}(z\sqrt{\epsilon/\alpha}) - \text{Erf}(h\sqrt{\epsilon/\alpha})]$; where G is the basal gradient.

Figure 1 shows a comparison between Equation 3, using a value of 5×10^{-5} for ϵ , and data taken in a borehole on the Greenland Ice Sheet (1,2). As the solution shows a stupendous resemblance to the observed data, one might wonder how long it has taken the Greenland Ice Sheet to reach this temperature distribution. Without troubling to solve the time dependent Equation 1, a person can estimate time rate of change through the characteristic times $1/\epsilon$, which measures how long it takes surface material to reach 2/3 of the way to the base of the glacier, and, h^2/α , which describes how long it takes 90% of a temperature change at a boundary to propagate by diffusion to the same depth. For the Greenland Ice Sheet the two values are 10,000yr and 50,000yr, respectively, which emphasizes the importance of convection in reaching a steady temperature rapidly. A value of 5×10^{-5} for ϵ is only about half of what a person would expect for the Greenland Ice Sheet, indicating that the observed temperatures shown in Figure 1 have not completely reached steady state.

References

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2/ Dahl-Jensen, D., Mosegaard, K., Gunderstrup, N., Clow, G.D., Johnson, S. J., Hansen, A.W., Balling, N. 1998. Past Temperatures Directly from the Greenland Ice Sheet. *Science*, **282**, 268-271.

Figure 1. Below. A comparison of theoretical and observed temperature in a borehole on the Greenland Ice Sheet. Small differences between the two are, most likely, vestiges of climate change at the glacier surface.

